Paper Reference(s) 9MA0/02
Pearson Edexcel Level 3 GCE

Mathematics

Advanced

PAPER 2: Pure Mathematics 2

Tuesday 11 June 2024 – Afternoon

Time: 2 hours

Question Booklet

DO NOT RETURN THIS BOOKLET WITH THE ANSWER BOOKLET.

Pearson

YOU MUST HAVE

Mathematical Formulae and Statistical Tables (Green), calculator

YOU WILL BE GIVEN

A separate Diagram Booklet
A separate Answer Booklet

INSTRUCTIONS

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the spaces provided in the Answer Booklet or in the separate Diagram Booklet – there may be more space than you need.

Do NOT write on this Question Booklet.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are 15 questions in this question booklet. The total mark for this paper is 100

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

There may be spare copies of some diagrams.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. $y = 4x^3 - 7x^2 + 5x - 10$

(a) Find in simplest form

(i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3 marks)

(b) Hence find the exact value of x

when $\dfrac{d^2y}{dx^2} = 0$

(2 marks)

(Total for Question 1 is 5 marks)

2. Jamie takes out an interest-free loan of £8100

Jamie makes a payment every month to pay back the loan.

Jamie repays £400 in month 1, £390 in month 2, £380 in month 3 and so on, so that the amounts repaid each month form an arithmetic sequence.

(a) Show that Jamie repays £290 in month 12
(1 mark)

(continued on the next page)

2. continued.

After Jamie's Nth payment, the loan is completely paid back.

(b) Show that $N^2 - 81N + 1620 = 0$
(2 marks)

(c) Hence find the value of N.
(2 marks)

(Total for Question 2 is 5 marks)

3. The point $P(3, -2)$ lies on the curve with equation $y = f(x)$, $x \in \mathbb{R}$

Find the coordinates of the point to which P is mapped when the curve with equation $y = f(x)$ is transformed to the curve with equation

(i) $y = f(x - 2)$

(ii) $y = f(2x)$

(iii) $y = 3f(-x) + 5$

(Total for Question 3 is 4 marks)

4. A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = ku_n - 5$$
$$u_1 = 6$$

where k is a positive constant.

Given that $u_3 = -1$

(a) show that

$$6k^2 - 5k - 4 = 0$$

(2 marks)

(continued on the next page)

4. continued.

(b) Hence

(i) find the value of k,

(ii) find the value of $\sum_{r=1}^{3} u_r$

(3 marks)

(Total for Question 4 is 5 marks)

5. Given that θ is small and in radians, use the small angle approximations to find an approximate numerical value of

$$\frac{\theta \tan 2\theta}{1 - \cos 3\theta}$$

(Total for Question 5 is 3 marks)

6. Look at the diagram for Question 6 in the separate Diagram Booklet.
The diagram shows a sketch of the curves with equations $y = f(x)$ and $y = g(x)$ where

$$f(x) = e^{4x^2} - 1 \qquad x > 0$$

$$g(x) = 8\ln x \qquad x > 0$$

(a) Find

(i) $f'(x)$

(ii) $g'(x)$

(2 marks)

(continued on the next page)

6. continued.

(b) Given that $f'(x) = g'(x)$ at $x = \alpha$

show that α satisfies the equation

$$4x^2 + 2\ln x - 1 = 0$$

(2 marks)

(continued on the next page)

6. continued.

(c) The iterative formula shown below,

$$x_{n+1} = \sqrt{\frac{1 - 2\ln x_n}{4}}$$

is used with $x_1 = 0{\cdot}6$ to find an approximate value for α

Calculate, giving each answer to 4 decimal places,

(i) the value of x_2

(ii) the value of α

(3 marks)

(Total for Question 6 is 7 marks)

7. **Look at the diagram for Question 7 in the separate Diagram Booklet.**
The diagram shows a sketch of the straight line l.

Line l passes through the points A and B.

(a) Relative to a fixed origin O

- **the point A has position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$**
- **the point B has position vector $5\mathbf{i} + 6\mathbf{j} + 8\mathbf{k}$**

Find $\overrightarrow{AB}$

(1 mark)

(continued on the next page)

7. continued.

(b) Given that a point P lies on l such that

$$|\overrightarrow{AP}| = 2|\overrightarrow{BP}|$$

find the possible position vectors of P.
(4 marks)

(Total for Question 7 is 5 marks)

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8. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\frac{1}{\operatorname{cosec}\theta - 1} + \frac{1}{\operatorname{cosec}\theta + 1} = 2\tan\theta\sec\theta$$

$\theta \neq (90n)°, \quad n \in \mathbb{Z}$

(3 marks)

(continued on the next page)

8. continued.

(b) Hence solve, for $0 < x < 90°$, the equation

$$\frac{1}{\operatorname{cosec} 2x - 1} + \frac{1}{\operatorname{cosec} 2x + 1} = \cot 2x \sec 2x$$

Give each answer, in degrees, to one decimal place.

(4 marks)

(Total for Question 8 is 7 marks)

9. Look at the diagram for Question 9 in the separate Diagram Booklet.
The diagram is a graph.
The graph shows the path of a small ball.

The ball travels in a vertical plane above horizontal ground.

The ball is thrown from the point represented by A and caught at the point represented by B.

The height, H metres, of the ball above the ground has been plotted against the horizontal distance, x metres, measured from the point where the ball was thrown.

(continued on the next page)

9. continued.

With respect to a fixed origin O, the point A has coordinates (0, 2) and the point B has coordinates (20, 0·8), as shown in the diagram.

The ball reaches its maximum height when $x = 9$

A quadratic function, linking H with x, is used to model the path of the ball.

(continued on the next page)

9. continued.

(a) Find H in terms of x.
(4 marks)

(b) Give one limitation of the model.
(1 mark)

(c) Chandra is standing directly under the path of the ball at a point 16 m horizontally from O.

Chandra can catch the ball if the ball is less than $2 \cdot 5$ m above the ground.

Use the model to determine if Chandra can catch the ball.
(2 marks)

(Total for Question 9 is 7 marks)

10. Look at the diagram for Question 10 in the separate Diagram Booklet.
The diagram shows a sketch of the curve C with parametric equations

$$x = (t + 3)^2$$

$$y = 1 - t^3$$

$$-2 \leq t \leq 1$$

(a) The point P with coordinates (4, 2) lies on C.

Using parametric differentiation, show that the tangent to C at P has equation

$$3x + 4y = 20$$

(5 marks)

(continued on the next page)

10. continued.

(b) The curve C is used to model the profile of a slide at a water park.

Units are in metres, with y being the height of the slide above water level.

Find, according to the model, the greatest height of the slide above water level.
(1 mark)

(Total for Question 10 is 6 marks)

11. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Look at the diagram for Question 11 in the separate Diagram Booklet.
The diagram shows a sketch of part of the curve C with equation

$$y = 8x^2e^{-3x} \qquad x \geq 0$$

The finite region R, shown shaded in the diagram, is bounded by:

- **the curve C**
- **the line with equation $x = 1$**
- **the x-axis**

(continued on the next page)

11. continued.

Find the exact area of R, giving your answer in the form

$$A + Be^{-3}$$

where A and B are rational numbers to be found.

(Total for Question 11 is 5 marks)

12. (a) Express $\dfrac{1}{V(25 - V)}$ in partial fractions.
(2 marks)

(b) The volume, V microlitres, of a plant cell t hours after the plant is watered is modelled by the differential equation given below.

$$\frac{dV}{dt} = \frac{1}{10}V(25 - V)$$

The plant cell has an initial volume of 20 microlitres.

Find, according to the model, the time taken, in minutes, for the volume of the plant cell to reach 24 microlitres.
(5 marks)

(continued on the next page)

Turn over

12. continued.

(c) Show that

$$V = \frac{A}{e^{-kt} + B}$$

where A, B and k are constants to be found.

(3 marks)

(d) The model predicts that there is an upper limit, L microlitres, on the volume of the plant cell.

Find the value of L, giving a reason for your answer.

(2 marks)

(Total for Question 12 is 12 marks)

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13. The world human population, P billions, is modelled by the equation

$$P = ab^t$$

where a and b are constants and t is the number of years after 2004

Using the estimated population figures for the years from 2004 to 2007, a graph is plotted of $\log_{10} P$ against t.

The points lie approximately on a straight line with:

- gradient 0·0054
- intercept 0·81 on the $\log_{10} P$ axis

(a) Estimate, to 3 decimal places, the value of a and the value of b.
(4 marks)

(continued on the next page)

13. continued.

(b) In the context of the model:

(i) interpret the value of the constant a,

(ii) interpret the value of the constant b.

(2 marks)

(c) Use the model to estimate the world human population in 2030

(2 marks)

(d) Comment on the reliability of the answer to part (c).

(1 mark)

(Total for Question 13 is 9 marks)

14. The circle C_1 has equation

$$x^2 + y^2 - 6x + 14y + 33 = 0$$

(a) Find:

(i) the coordinates of the centre of C_1

(ii) the radius of C_1

(3 marks)

(continued on the next page)

14. continued.

(b) A different circle C_2

- **has centre with coordinates $(-6, -8)$**
- **has radius k, where k is a constant**

Given that C_1 and C_2 intersect at 2 distinct points,

find the range of values of k, writing your answer in set notation.
(5 marks)

(Total for Question 14 is 8 marks)

15. The curve C has equation

$$(x + y)^3 = 3x^2 - 3y - 2$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of x and y.

(5 marks)

(b) The point P(1, 0) lies on C.

Show that the normal to C at P has equation

$$y = -2x + 2$$

(2 marks)

(continued on the next page)

Turn over

15. continued.

(c) Prove that the normal to C at P does NOT meet C again.

You should use algebra for your proof and make your reasoning clear.
(5 marks)

(Total for Question 15 is 12 marks)

END OF PAPER

TOTAL FOR PAPER IS 100 MARKS